Effective Conductivity and Microwave Reflectivity of Thin Metallic Films

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Abstract —**Thin metallic films have reduced conductivity when thickness is not larger than electron mean free path, a phenomenon studied by Lord Thompson. Formulas given by Liao are valid for small thickness; a general formula is available for all ranges of film thickness. This result is in terms of an exponential integral which is easily computed, and is graphed. A simple and accurate formula for calculating microwave reflection from a thin metallic film is developed. The equivalent circuit has the film surface resistivity in shunt with 120 π (or the substrate impedance). Examples of surface resistivity and reflection coefficient for a gold film are given graphically.**

I. INTRODUCTION

Thin metallic films deposited upon dielectric substrates or radomes are sometimes used for EMP and precipitation static protection, for defogging, deicing, etc. When the film thickness is large compared with the mean free path of electrons in the metal, the bulk conductivity applies. Smaller thicknesses experience a decrease in conductivity due to the scattering of electrons from the film surface. Liao [1], [2] gives a formula for effective conductivity which is valid only for film thickness small with respect to mean free path. This communication gives calculations based upon an exact formula. Also given is a simple formula for calculating the reflection coefficient for a plane-wave incident upon a thin metallic film.

II. EFFECTIVE CONDUCTIVITY OF A THIN FILM

Thompson, in 1901, analyzed mean free paths inside a thin film by assuming diffuse scattering independent of angle by electrons incident on the surface from within the film. These paths were integrated to obtain an equivalent mean free path, from which the effective conductivity could be calculated. His result was improved by Fuchs [3], Sondheimer [4], and Campbell [5] who considered also the electrons originating at the surface. They kept the diffuse scattering assumption and integrated the Boltzmann equation in one dimension to obtain an integral for the ratio of effective conductivity σ to bulk conductivity σ_0 in terms of the ratio of film thickness t to bulk electron mean free path length p with $x = t/p$

$$\frac{\sigma}{\sigma_0} = 1 - \frac{3}{2x}\int_1^\infty \left[\frac{1}{q^3} - \frac{1}{q^5}\right]\left[1 - e^{-xq}\right] dq. \tag{1}$$

Ramey and Lewis [6] also used the Sondheimer result. The integration variable is $q = 1/\cos\theta$ as the velocity is written in polar coordinates. This integral can be integrated exactly in terms of the Exponential Integral $E_1(x)$; see Abramowitz and Stegun [7]

$$\frac{\sigma}{\sigma_0} = 1 - \frac{3}{8x} + \frac{e^{-x}}{16x}(6 - 10x - x^2 + x^3) + \frac{x}{16}(12 - x^2)E_1(x). \tag{2}$$

Various approximations have been given by the authors quoted, and in fact Thompson's result is an approximation for $t \ll p$. However, most computer libraries have an Exponential Integral subroutine, and Abramowitz and Stegun have an excellent table, so that it is not necessary to use approximations. Fig. 1 shows σ/σ_0 versus x for a three decade range of x. Note that roughly $t = 3p$ is needed to achieve 90 percent of the bulk conductivity, while $t = 0.1$ gives a conductivity roughly 5 times poorer than bulk.

This result is applied to a gold film, with $\sigma_0 = 4.1 \times 10^7$ mho/m and $p = 0.057$ μm. Fig. 2 shows surface resistivity R_s versus film thickness for gold, over a range of R_s from 100 $\Omega/\square$ to 0.1 $\Omega/\square$. When the thin film thickness is small with respect to the skin depth δ, the surface resistivity is just

$$R_s = \frac{1}{\sigma t}. \tag{3}$$

Experimental evidence indicates that this model fits polycrystalline films reasonably well [5]. Single crystal films, however, do not fit the Fuchs–Sondheimer–Campbell theory. An improved theory assumes diffuse electron scattering at the surface when the incidence is near normal, and specular reflection when the incidence is highly oblique [8]. Another refinement of the theory introduces the shape and fine structure of the energy surface [8]. Since most thin films will be polycrystalline, the results given here should be useful.

III. REFLECTION OF PLANE WAVES BY A THIN FILM

A plane wave normally incident on a thin film can be represented by a voltage applied to a transmission line which is terminated in an impedance $\eta = 120$ π. If the film is deposited upon a dielectric substrate, the termination can be replaced by a transmission line representing the dielectric sheet. When oblique incidence occurs, the same equivalent circuit can be used, with the transmission line parameters changed: the normal propagation constant is multiplied by $\cos\theta$ where θ is the incidence angle measured from normal; the normal characteristic impedance is multiplied by $\cos\theta$ for parallel polarization, and divided by $\cos\theta$ for normal polarization [9]. Input impedance for the normally incident plane wave is

$$Z_{in} = Z_0 \frac{\eta + Z_0 \tanh\gamma t}{Z_0 + \eta\tanh\gamma t} \tag{4}$$

where t is the film thickness as before and

$$Z_0 = \sqrt{j\omega\mu/(\sigma + j\omega\epsilon)} \qquad \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}. \tag{5}$$

For metals and semiconductors $\sigma \gg \omega\epsilon$ so the transmission line parameters may be simplified. It is illuminating to write γt in terms of skin depth δ

$$\gamma t = (1 + j)t/\delta. \tag{6}$$

If $t \gg \delta$ the input impedance reduces to Z_0 with the well known result $Z_{in} = (1 + j)/\sigma\delta = (1 + j)R_s$. Of more interest here is $t \ll \delta$ for which the input impedance becomes

$$Z_{in} \simeq \eta \frac{1 + jkt}{1 + \eta\sigma t} = \frac{R_s \eta}{R_s + \eta} \tag{7}$$

where for the thin film $R_s = 1/\sigma t$. Note that the skin depth has been replaced by film thickness. Thus, as pointed out by Hawthorne [10], the surface resistance is simply in shunt with η. A dielectric substrate with its transmission line section can now have this shunt load, if the incident wave sees the dielectric first, or R_s is in shunt with the transmission line input, if the metallic film is first.

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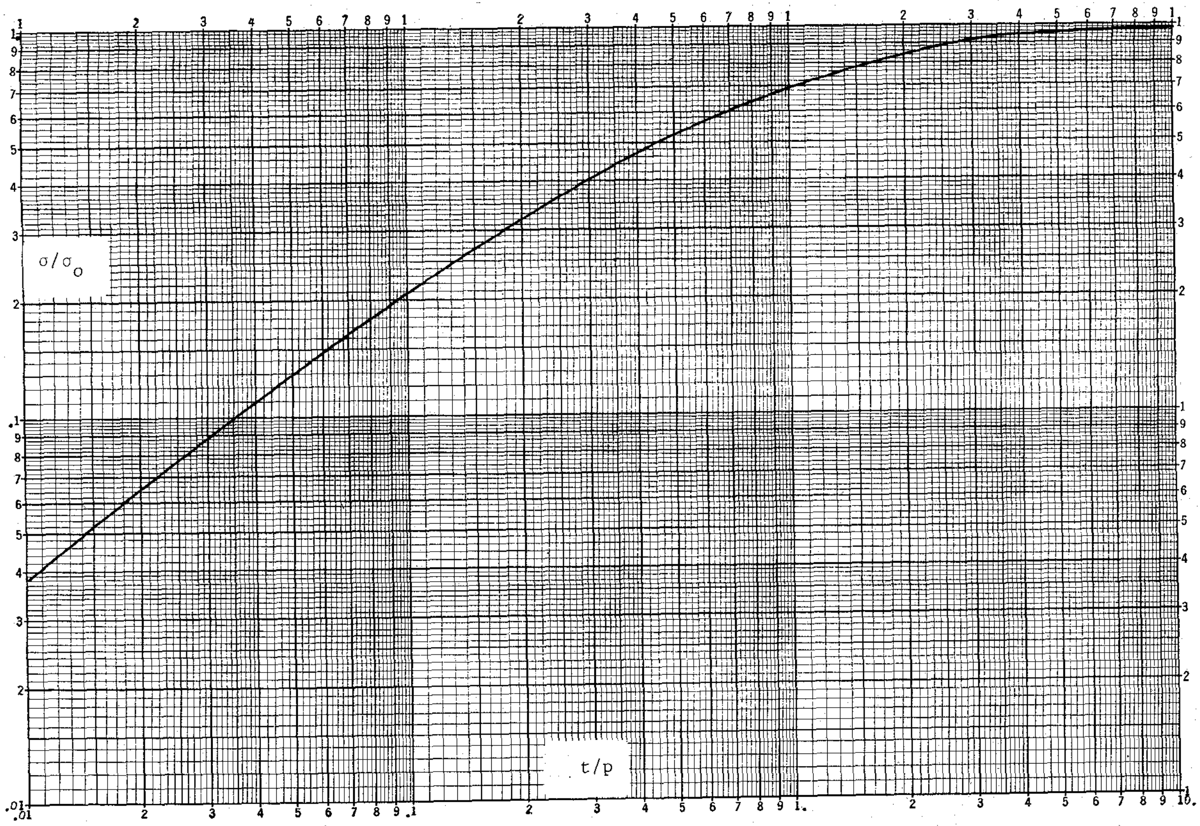

Fig. 1. Effective conductivity versus thickness to electron mean free path ratio.

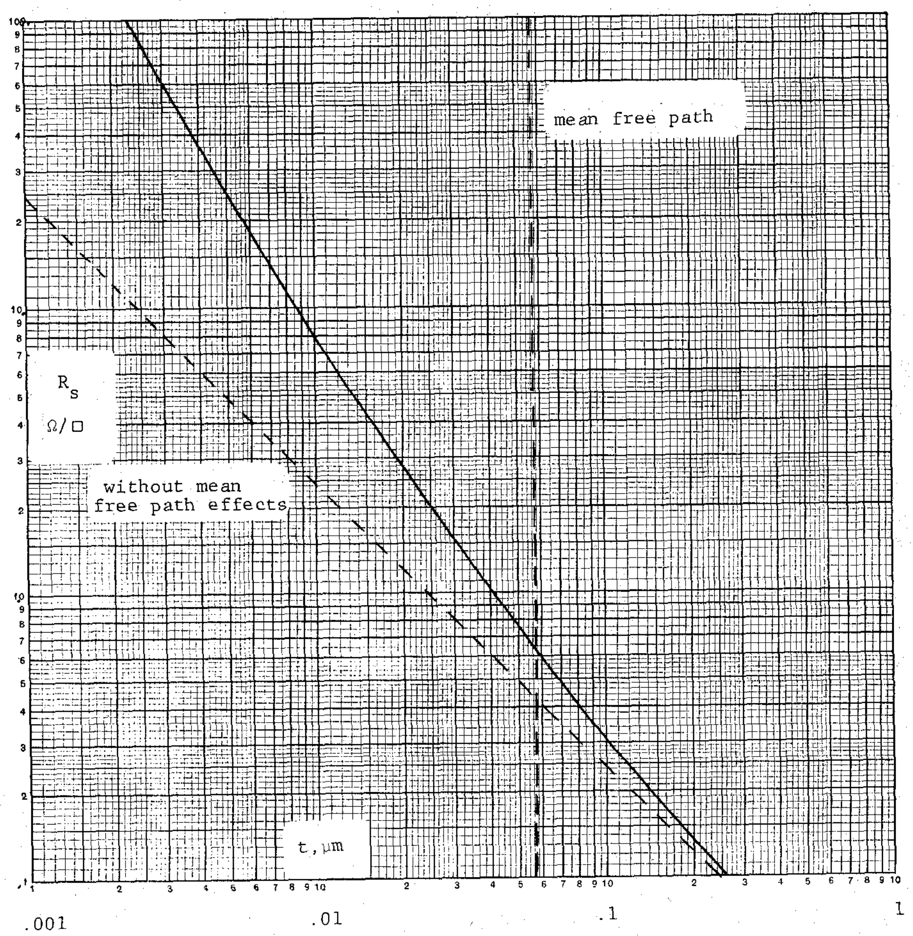

Fig. 2. Gold film surface resistivity.

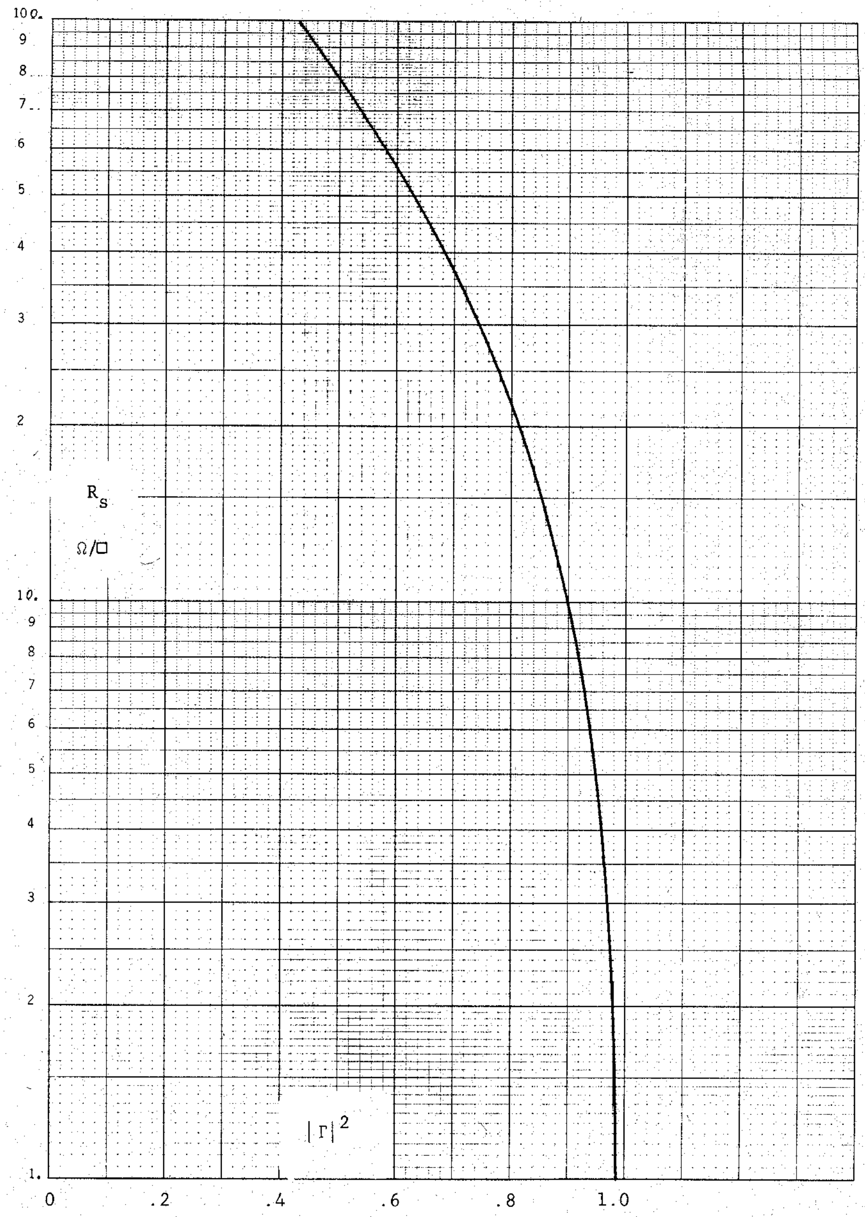

Fig. 3. Power reflection coefficient for thin film.

For the film alone, the reflection coefficient is

$$\Gamma = -\frac{\eta}{2R_s+\eta}\ (Z_0=\eta). \tag{8}$$

This is valid for $t \ll \delta$ and is more direct than the shielding formulas used by Liao; the more precise formula above can be used where t is thicker. Of course, when $t < p$ the effective conductivity must be used in all these calculations due to the reduced electron mean free path. Fig. 3 shows the power reflection coefficient $|\Gamma|^2$ for a thin film in terms of R_s. These data can be used to quickly calculate thin metallic film performance.

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